MAGNETIC PHOTONIC CRYSTALS AS ARTIFICIAL MAGNETOELECTRICS

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Abstract Magnetic photonic crystals are spatially periodic dielectric composites with at least one of the constitutive components being a magnetically polarized material. We show that the electrodynamic properties of magnetic photonic crystals with proper configuration correspond to those of hypothetical media with huge linear magnetoelectric effect. In particular, such composites can display strong asymmetry $\omega(\boldsymbol{k}) \neq \omega(-\boldsymbol{k})$ of the electromagnetic dispersion relation, which can result in a number of interesting phenomena, including the electromagnetic unidirectionality. A unidirectional medium, being perfectly transmissive for electromagnetic wave of certain frequency, freezes the radiation of the same frequency propagating in the opposite direction in the form of a coherent standing wave with zero group velocity. At microwave frequencies, unidirectional photonic crystals can be made of common ferro- or ferrimagnetic materials alternated with anisotropic dielectric layers.

Keywords: Artificial magnetoelectrics, magnetic photonic crystals, electrodynamics of nonreciprocal media, frozen mode.

1. Electrodynamics of magnetoelectric media

Electrodynamics of magnetoelectric media can be described by the standard time-harmonic Maxwell equations

$$\nabla \times \boldsymbol{E} = \frac{i\omega}{c}\boldsymbol{B}, \quad \nabla \times \boldsymbol{H} = -\frac{i\omega}{c}\boldsymbol{D}, \tag{1}$$

with linear material relations

$$\boldsymbol{D} = \varepsilon(\omega)\boldsymbol{E} + \chi(\omega)\boldsymbol{H}, \quad \boldsymbol{B} = \mu(\omega)\boldsymbol{H} + \chi^T(\omega)\boldsymbol{E}. \tag{2}$$

where $\boldsymbol{\varepsilon}(\omega)$ and $\boldsymbol{\mu}(\omega)$ are the electric permittivity and magnetic permeability tensors, $\boldsymbol{\chi}(\omega)$ is the tensor of linear magnetoelectric response. Unlike $\boldsymbol{\varepsilon}(\omega)$ and $\boldsymbol{\mu}(\omega)$, the tensor $\boldsymbol{\chi}(\omega)$ is odd with respect to time reversal R and space

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inversion I. This implies that $\chi = 0$ in all nonmagnetic and/or centrosymmetric media [1,2]. More specifically

$$\chi \neq 0, \text{ only if } R \notin G, \ I \notin G \tag{3}$$

where G is magnetic symmetry group of the medium.

A remarkable manifestation of linear magnetoelectric effect is the phenomenon of electromagnetic spectral asymmetry

$$\omega(\boldsymbol{k}) \neq \omega(-\boldsymbol{k}) \tag{4}$$

which can occur exclusively in magnetoelectric media (see, for example, [3], and references therein). For a particular direction of the wave vector $\boldsymbol{k}$, the spectral asymmetry (4) can occur only if none of the symmetry operations from G reverses the direction of $\boldsymbol{k}$

$$\omega(\boldsymbol{k}) \neq \omega(-\boldsymbol{k}) \text{ only if } g\boldsymbol{k} \neq -\boldsymbol{k} \ \text{ for all } g \in G. \tag{5}$$

Obviously, if the symmetry group G includes time reversal and/or space inversion, the criterion (5) cannot be satisfied

$$\text{if } R \in G \ \text{ or } I \in G, \ \text{ then } \omega(\boldsymbol{k}) = \omega(-\boldsymbol{k}) \text{ for any } \boldsymbol{k}. \tag{6}$$

Nor can such a medium display the linear magnetoelectric effect.

For the majority of dielectric materials, the linear magnetoelectric effect is prohibited by symmetry. But even in those cases where it is allowed, the magnitude of the effect is very small. In known magnetoelectric crystals the magnitude of nonzero components of magnetoelectric tensor χ does not exceed $10^{-3} - 10^{-4}$ [2]. As a consequence, the remarkable properties of magnetoelectrics featuring electromagnetic spectral asymmetry (4) have not found any significant application. In addition, natural magnetoelectric crystals often have complicated and unpredictable domain structure that further suppresses their nonreciprocal properties and makes them unattractive for practical use. Our objective here is to introduce nonreciprocal dielectric composites, known as magnetic photonic crystals, as an alternative to natural magnetoelectric materials. Although nonreciprocal photonic crystals do not display static magnetoelectric effect, their electrodynamics is similar to that of hypothetical magnetoelectric materials with highly enhanced nonreciprocal properties. In particular, they can display extremely strong asymmetry (4) of electromagnetic dispersion relation, which is unachievable in any natural homogeneous material. The strong spectral asymmetry, in turn, can result in the unique phenomenon of electromagnetic unidirectionality [4,5] when electromagnetic waves can propagate through the medium only in one of the two opposite directions.

2. Symmetry of magnetic photonic crystals

Photonic crystals are spatially periodic arrays of two or more different dielectric components. In magnetic photonic crystals, at least one of the constituents is a magnetically polarized material (see, for example, [8] and references therein). At the frequency range of interest, all the constitutive components are presumed electromagnetically lossless. As a consequence of spatial periodicity, the electromagnetic frequency spectrum of a photonic crystal develops a band-gap structure similar to that of electrons in a crystal lattice [7]. We assume that each of the constitutive components of photonic crystal is a uniform dielectric material satisfying conventional constitutive relations

$$\boldsymbol{D} = \varepsilon(\omega)\boldsymbol{E}, \quad \boldsymbol{B} = \mu(\omega)\boldsymbol{H} \tag{7}$$

with Hermitian material tensors

$$\varepsilon^{\dagger}(\omega) = \varepsilon(\omega), \quad \mu^{\dagger}(\omega) = \mu(\omega). \tag{8}$$

The property (8) of Hermitivity implies electromagnetic losslessness of the medium. The tensors $\boldsymbol{\varepsilon}(\omega)$ and $\boldsymbol{\mu}(\omega)$ are different in different components of the periodic array. The absence of magnetoelectric terms in (7) implies that each uniform constituent, if it fills the entire space, has perfectly symmetric electromagnetic dispersion relation

$$\omega(\boldsymbol{k}) = \omega(-\boldsymbol{k}), \tag{9}$$

which is the case with all non-magnetic and overwhelming majority of magnetic materials. At the same time we expect that spatially periodic array of such "non-magnetoelectric" components can support essentially asymmetric electromagnetic spectrum. In other words, in magnetic photonic crystals, the property (4) of bulk spectral asymmetry can be achieved by proper space arrangement of constitutive components, rather than by incorporating magnetoelectric materials [4].

From symmetry standpoint, photonic crystals, being spatially periodic, can be viewed as artificial macroscopic crystals. Therefore, every photonic crystal can be assigned certain magnetic symmetry group G, which along with rotations, reflections, and translations may also include time reversal operation R combined with some space transformations [1]. Knowing magnetic symmetry G of the periodic array, one can apply the criterion (5) to find out whether or not one can expect asymmetric dispersion relation for a particular direction of the wave vector $\boldsymbol{k}$. This can only occur if the symmetry group G is on the list of those compatible with linear magnetoelectric effect [1]. It does not mean, though, that the magnetic photonic crystal can display any static magnetoelectric effect.

At first sight, the problem of assigning magnetic symmetry group G to a photonic crystal seems to be quite straightforward. Indeed, knowing the geometry of the periodic array and the symmetry G_i^0 of each individual constitutive component, one can immediately obtain the exact magnetic symmetry G^0 of the photonic crystal. The so obtained symmetry group G^0 will be referred to as the *true symmetry group* of photonic crystal. By definition, the periodic array is invariant under operations from the true symmetry group G^0.

The important point, though, is that the symmetry of the Maxwell equations (1) together with the constitutive relations (7) can be higher than G^0. Indeed, as far as electrodynamics is concerned, each constitutive material of the periodic array is represented by the respective material tensors $\boldsymbol{\varepsilon}(\omega)$ and $\boldsymbol{\mu}(\omega)$ in the relations (7). For a particular constitutive component *i*, the symmetry G_i of the respective material tensors $\boldsymbol{\varepsilon}(\omega)$ and $\boldsymbol{\mu}(\omega)$ can be higher than the symmetry G_i^0 of the material itself. For instance, both $\boldsymbol{\varepsilon}(\omega)$ and $\boldsymbol{\mu}(\omega)$, being second rank tensors, are always centrosymmetric regardless of whether or not the material itself supports space inversion. The above argument shows that the symmetry group *G* that describes the electrodynamics of photonic crystal can be higher compared to its true symmetry group G^0. Hereinafter, the group *G* will be referred to as the *electromagnetic symmetry group*. Obviously, $G^0 \subseteq G$.

If indeed the electromagnetic symmetry group *G* appears to be higher than the true symmetry group G^0, one can expect the situation where a particular effect, such as spectral asymmetry, is prohibited by *G* but allowed by G^0. This situation implies that although this particular effect can occur, it is associated with physical processes unaccounted for by the Maxwell equations (1) with conventional constitutive relations (7). All such interactions and effects are presumed insignificant. They may include, but are not limited to: electrostriction and/or magnetostriction, space dispersion (e.g., reciprocal optical activity of noncentrosymmetric materials [1]), surface effects at the interfaces between different components of the photonic crystal, magnetoelectric effect in constitutive materials (if any). Hereinafter, we will focus exclusively on the robust bulk electrodynamic effects which are accounted for by the Maxwell equations (1) with the conventional constitutive relations (7). Thus, our symmetry consideration will be based on the electromagnetic symmetry group *G*, rather than on its subgroup G^0. Note that in many cases the two symmetries are simply identical ($G \equiv G^0$). An example to the contrary is a photonic crystal with a ferroelectric constitutive component.

A photonic crystal can display electromagnetic spectral asymmetry (4) only if its symmetry group *G* includes neither time reversal nor space inversion. If none of the constitutive components of a photonic crystal supports any kind of spontaneous magnetic order, nor is an external magnetic field applied, then the photonic crystal certainly possesses time reversal symmetry *R* and supports perfectly symmetric dispersion relation (9). Thus, asymmetric dispersion relation can be found exclusively in magnetic photonic crystals. The distinguishing feature of the material tensors $\boldsymbol{\varepsilon}(\omega)$ and $\boldsymbol{\mu}(\omega)$ in magnetically polarized media is that both tensors are complex

$$\varepsilon^*(\omega) = \varepsilon(-\omega) \neq \varepsilon(\omega), \quad \mu^*(\omega) = \mu(-\omega) \neq \mu(\omega),$$

while in lossless nonmagnetic media the tensors $\boldsymbol{\varepsilon}(\omega)$ and $\boldsymbol{\mu}(\omega)$ are real and symmetric

$$\boldsymbol{\varepsilon}^*(\omega) = \boldsymbol{\varepsilon}(-\omega) = \boldsymbol{\varepsilon}(\omega), \ \ \boldsymbol{\mu}^*(\omega) = \boldsymbol{\mu}(-\omega) = \boldsymbol{\mu}(\omega).$$

In homogeneous media, the imaginary (skew-symmetric) parts of $\boldsymbol{\varepsilon}(\omega)$ and $\boldsymbol{\mu}(\omega)$ are responsible for the nonreciprocal effect of Faraday rotation, while in periodic heterogeneous media the same terms can also cause the effect (4) of electromagnetic spectral asymmetry. In fact, the degree of electromagnetic spectral asymmetry is directly related to the magnitude of Faraday rotation in the magnetic constituent of photonic crystal. Note that in the static limiting case $\omega = 0$, the material tensors $\boldsymbol{\varepsilon}(\omega)$ and $\boldsymbol{\mu}(\omega)$ become real and symmetric, implying that all electromagnetic nonreciprocal effects vanish as $\omega \rightarrow 0$. By contrast, in natural magnetoelectric crystals, where the electromagnetic spectral asymmetry is associated with the tensor χ in Eq (2), the magnetoelectric effect persists even if $\omega \rightarrow 0$, although it is extremely small at all frequencies.

Unlike the situation with time reversal symmetry R, the space inversion I is always supported by both material tensors $\boldsymbol{\varepsilon}(\omega)$ and $\boldsymbol{\mu}(\omega)$ in every uniform constitutive component of the composite structure, regardless of presence or absence of magnetic and/or electric polarization. To remove space inversion from electromagnetic symmetry group G of the periodic array and, thereby, to allow for electromagnetic spectral asymmetry, one has to choose a proper spatial arrangement of the constitutive components. To put it differently, the structural geometry of the photonic crystal must be complex enough not to support space inversion.

To sum up, we can state that only magnetic photonic crystals with special geometry can support asymmetric electromagnetic dispersion relation (4). The criterion (5) is just a necessary condition for spectral asymmetry. Even if this condition is met, the effect of spectral asymmetry may appear to be negligible or even ruled out by physical reasons different from those imposed by magnetic symmetry. To find out if a photonic crystal satisfying the criterion (5) does display the electromagnetic spectral asymmetry, one has to go beyond the symmetry consideration and deal with the Maxwell equations (1) in the heterogeneous medium. Several specific examples are considered in the nest section.

3. Nonreciprocal periodic stacks

Photonic crystals can have one-, two- or three-dimensional periodicity. One-dimensional photonic crystals are commonly referred to as periodic stacks, or multilayers. The symmetry arguments based on the criterion (5) for spectral asymmetry can be applied with equal ease to photonic crystals of any dimensionality. But if we want to go further and actually solve the Maxwell equations in the composite medium, then the case of one-dimensional periodicity is the most attractive. On the other hand, magnetic multilayers appear to be the most practical composites supporting strong electromagnetic spectral asymmetry (4). Therefore, in further consideration we will focus exclusively on periodic magnetic stacks.

Let us start with periodic stacks with just two different layers in a unit cell. Electromagnetic symmetry group G of such a periodic array always supports space inversion symmetry with the center of inversion in the middle of each uniform layer. Therefore, a periodic stack composed of two alternating layers will never display electromagnetic spectral asymmetry, regardless of the materials of the layers. Let us reiterate that referring to the electromagnetic symmetry group G rather than to the true magnetic symmetry group G^0 of the photonic crystal, we disregard those presumably insignificant effects which cannot be accounted for using the time-harmonic Maxwell equation (1) with the conventional constitutive relations (7). In many cases, though, the symmetry groups G and G^0 are simply identical.

A periodic stack with asymmetric bulk dispersion relation

Consider magnetic periodic stacks with three layers in a unit cell. In this case there is a possibility of removing space inversion from the electromagnetic symmetry group G of the periodic array. A simple example of the kind is shown in Fig . 1.

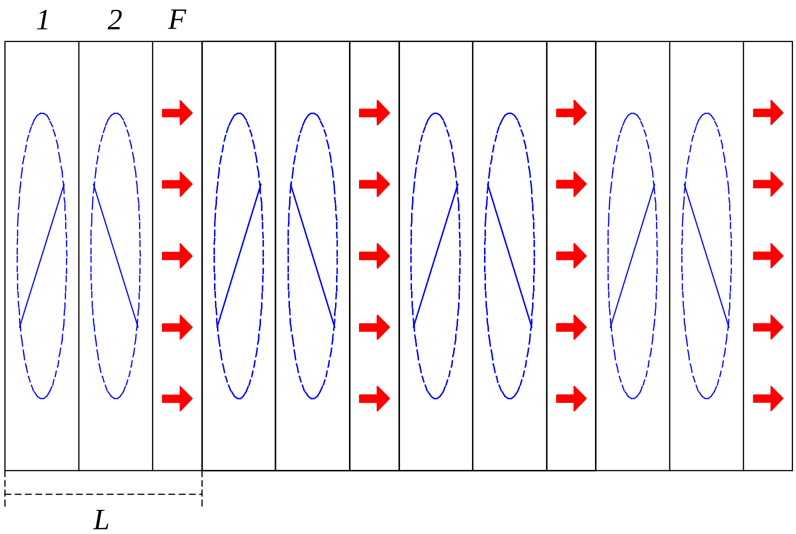

Figure 1. A simplest periodic magnetic stack supporting asymmetric bulk dispersion relation. A unit cell L of this stack comprises three layers: two anisotropic dielectric layers 1 and 2 with misaligned in-plain anisotropy (the A - layers), and one magnetic layer F with magnetization shown by the arrows.

The F – layers in Fig. 1 are ferromagnetic with magnetization parallel to the z - direction. The respective material tensors are

$$\boldsymbol{\varepsilon}_F = \begin{bmatrix} \varepsilon_F & i\alpha & 0 \\ -i\alpha & \varepsilon_F & 0 \\ 0 & 0 & \varepsilon_F' \end{bmatrix}; \ \boldsymbol{\mu}_F = \begin{bmatrix} \mu_F & i\beta & 0 \\ -i\beta & \mu_F & 0 \\ 0 & 0 & \mu_F' \end{bmatrix} \tag{10}$$

The real parameters α and β are responsible for circular birefringence (Faraday rotation), both of them are odd functions of frequency ω. At frequencies below 10^{12} Hz, the dominant contribution to the Faraday rotation usually comes from the "magnetic" parameter β, which can become particularly large in the vicinity of magnetic resonance. The periodic array in Fig. 1 can display strong spectral asymmetry only if either of the two gyrotropic parameters α and β is large enough. Specifically, at least one of the two quantities α/ε_F or β/μ_F must be larger than 10^{-1}.

For simplicity, the *A* - layers are presumed nonmagnetic

$$\boldsymbol{\varepsilon}_A = \begin{bmatrix} \varepsilon_A + \delta\cos 2\phi & \delta\sin 2\phi & 0 \\ \delta\sin 2\phi & \varepsilon_A - \delta\cos 2\phi & 0 \\ 0 & 0 & \varepsilon_A' \end{bmatrix}, \tag{11}$$

$$\boldsymbol{\mu}_A = \begin{bmatrix} \mu_A + \Delta\cos 2\phi & \Delta\sin 2\phi & 0 \\ \Delta\sin 2\phi & \mu_A - \Delta\cos 2\phi & 0 \\ 0 & 0 & \mu_A' \end{bmatrix}.$$

The term "nonmagnetic" implies that both material tensors are real and symmetric. Parameters δ and Δ describe the in-plane anisotropy, while the angle ϕ defines the orientation of the principle axes of the tensors $\boldsymbol{\varepsilon}(\omega)$ and $\boldsymbol{\mu}(\omega)$ in the *x*-*y* plane. All *A* - layers are made of the same dielectric material and have the same thickness. The only parameter that may differ in different *A* - layers is the orientation ϕ. Another necessary condition for the periodic array in Fig. 1 to display strong spectral asymmetry is that the in-plane dielectric anisotropy of the *A* - layers is strong enough. Specifically, at least one of the two quantities δ/ε_A and/or Δ/μ_A is significant (10^{-1} or larger).

All essentially different periodic arrays of the *A* - and *F* - layers with three layers in a primitive cell are equivalent to a single one shown in Fig. 1. A primitive cell comprises one *F* - layers and two *A* - layers with different orientations ϕ_1 and ϕ_2. The most critical parameter of this structure is the misalignment angle $\phi = \phi_1 - \phi_2$ between the adjacent *A* - layers. This angle determines the electromagnetic symmetry group *G* of the stack, along with the symmetry of its electromagnetic dispersion relation. The results are summarized in the following table

Table 1.

Misalignment angle	Magnetic symmetry	Spectral symmetry
$\phi = 0$	$m'm'm$	$\omega(\boldsymbol{k}) = \omega(-\boldsymbol{k})$ for all $\boldsymbol{k}$
$\phi = \pi/2$	$\bar{4}m'm'$	$\omega(\boldsymbol{k}) = \omega(-\boldsymbol{k})$ for $\boldsymbol{k} \parallel z$
$\phi \neq 0,\ \pi/2$	$2'2'2$	$\omega(\boldsymbol{k}) \neq \omega(-\boldsymbol{k})$ for $\boldsymbol{k} \parallel z$

Note that in the case $\phi = 0$, the three layered unit cell in Fig. 1 reduces to a two layered cell with doubled thickness of the *A* - layer. As we already know, the electromagnetic symmetry group of a periodic stack with two-layered unit cell always supports space inversion and, therefore, displays symmetric dispersion relation $\omega(\boldsymbol{k}) = \omega(-\boldsymbol{k})$ for an arbitrary direction of the wave vector $\boldsymbol{k}$, regardless of the materials of the layers.

Typical numerical example of electromagnetic dispersion relations of the nonreciprocal periodic array in Fig. 1 is shown in Fig. 2.

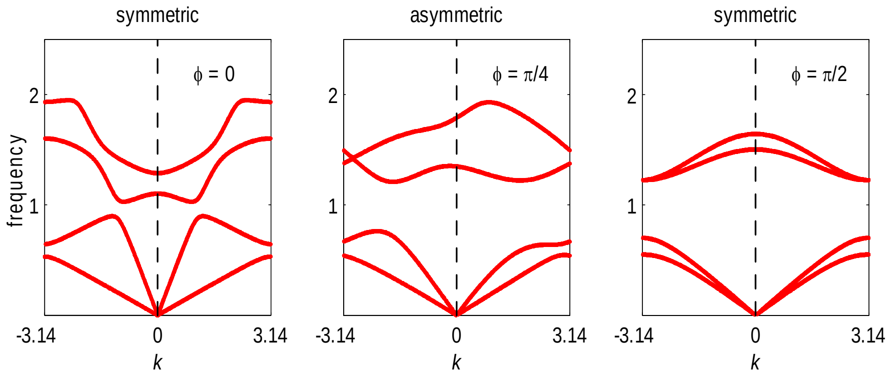

Figure 2. Example of electromagnetic dispersion relations $\omega(k)$ of the nonreciprocal periodic stack in Fig. 1. Three graphs correspond to three different values of the misalignment angle ϕ between adjacent *A* - layers.

In accordance with the Table 1, the spectral asymmetry (4) for $\boldsymbol{k} \parallel z$ exists only if the misalignment angle ϕ is not a multiple of $\pi/2$.

The above example presents the simplest and the most symmetric periodic stack supporting the bulk spectral asymmetry. More examples can be found in Ref. [4].

4. Electromagnetic unidirectionality

Strong electromagnetic spectral asymmetry has various physical consequences, one of which is the effect of *unidirectional wave propagation*. Suppose that at $k = k_0$, $\omega = \omega_0$, one of the spectral branches $\omega(\boldsymbol{k})$ develops a stationary inflection point

$$\omega_k'(k_0) = 0; \quad \omega_{kk}''(k_0) = 0; \quad \omega_{kkk}'''(k_0) \neq 0, \tag{13}$$

as shown in Fig. 3(a).

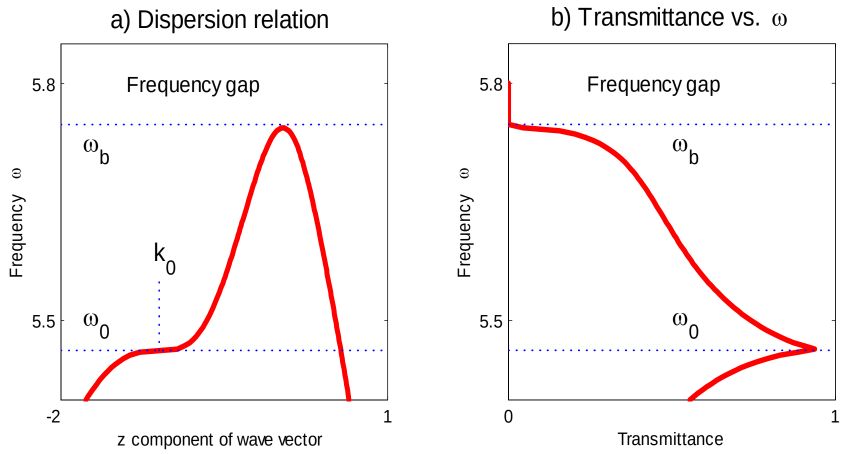

Figure 3. (a) A fragment of asymmetric dispersion relation $\omega(k)$ of the periodic stack shown in Fig. 1. At $k = k_0$ and $\omega = \omega_0$ this spectral branch develops a stationary inflection point associated with electromagnetic unidirectionality and the frozen mode. ω_b is the edge of the frequency band. (b) The respective transmittance τ of the nonreciprocal semi-infinite slab vs. frequency. At the frequency ω_0 of stationary inflection point, τ is close to unity, which implies that the incident wave is almost completely converted into the frozen mode with zero group velocity and gigantic amplitude. The values of ω and k are expressed in units of c/L and $1/L$, respectively.

With certain reservations, the energy velocity of electromagnetic wave coincides with its group velocity $u(k) = \omega_k'(k)$. At frequency $\omega = \omega_0$ there are two Bloch waves: one with $k = k_0$ and the other with $k = k_1$. Obviously, only one of the two waves can transfer electromagnetic energy – the one with $k = k_1$ and the group velocity $u(k_1) < 0$. The Bloch eigenmode with $k = k_0$ has zero group velocity $u(k_0) = 0$ and does not transfer energy. This latter eigenmode associated with stationary inflection point (13) is referred to as the *frozen mode*. As one can see in Fig. 3(a), none of the two eigenmodes with $\omega = \omega_0$ has positive group velocity and, therefore, none of the electromagnetic eigenmodes can transfer energy from left to right at this particular frequency! Thus, a photonic crystal with the dispersion relation similar to that in Fig. 3(a), displays the property of *electromagnetic unidirectionality* at $\omega = \omega_0$. Such a remarkable effect can be viewed as an extreme manifestation of the spectral asymmetry (4).

The effect of electromagnetic unidirectionality can occur in magnetic photonic crystals made up of common dielectric and ferro- or ferrimagnetic components (at least at frequencies below 10^{12} Hz). There are two key physical requirements for that:

1) The electromagnetic symmetry group G of the periodic array must be compatible with the criterion (5) for spectral asymmetry.
2) The magnetic constituent must display significant circular birefringence at frequency range of interest (at least 10%, or more).
3) The anisotropic layers must display significant in-plane anisotropy (at least 10%, or more).

Failure to satisfy the conditions 2 and/or 3 does not formally rule out the phenomenon of unidirectionality, but it would obscure the effect. Indeed, weak Faraday rotation or weak anisotropy leads to a small value of the third derivative $\omega_k'''(k)$ in (13), which, in turn, pushes the stationary inflection point ω_0 in Fig. 3(a) too close to the photonic band edge ω_b.

Electromagnetic properties of a semi-infinite unidirectional slab

Consider a plane electromagnetic wave propagating from left to right and impinging on the boundary of a semi-infinite unidirectional photonic slab with dispersion relation shown in Fig. 3(a). Due to spectral asymmetry, the situation of the right-to-left propagation appears to be quite different and will not be discussed here (see the details in [5]). At the slab boundary, a portion of the incident wave is reflected back and the rest enters the semi-infinite slab. Let $S_I > 0$, $S_R < 0$, and $S_T > 0$ be the energy flux of the incident, reflected and transmitted waves, respectively. Due to the energy conservation, $S_I + S_R = S_T$. The transmittance (τ) and reflectance (ρ) of the semi-infinite slab are defined as

$$\tau = \frac{S_T}{S_I}, \quad \rho = -\frac{S_R}{S_I}. \tag{14}$$

The energy conservation implies that $\rho = 1 - \tau$.

In the case of a single propagating mode, the transmitted energy flux S_T inside the slab can be expressed in terms of the mode energy density W_T and its group velocity $u(k)$

$$S_T = u(k)\ W_T. \tag{15}$$

According to Eq (13) and Fig. 3(a), the group velocity $u(k)$ of the transmitted wave vanishes as $\omega \to \omega_0$ and $k \to k_0$. At the same time, the transmittance τ along with the energy flux S_T remains finite even at $\omega = \omega_0$, as illustrated in Fig. 3(b). This implies that the electromagnetic field amplitude inside the unidirectional slab increases dramatically in the vicinity of the frozen mode frequency ω_0

$$W_T \sim |\omega - \omega_0|^{-2/3}, \text{ as } \omega \to \omega_0, \tag{16}$$

while the wave slows down. In fact, the incident electromagnetic wave with frequency close to ω_0 gets trapped inside the slab in the form of coherent frozen mode with huge amplitude and nearly zero group velocity. Detailed mathematical analysis of such a remarkable phenomenon is carried out in [5,6].

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